Lecture 6 - Sep. 24

Math Review

Incompleteness of Postcondition
Interpreting the choose operator
Cross Product
Relation

Announcements/Reminders

- Lab1 solution released
- Lab2 released

Bidirectional Subset Relations: Programming

$S = T \iff S \subseteq T \land T \subseteq S$

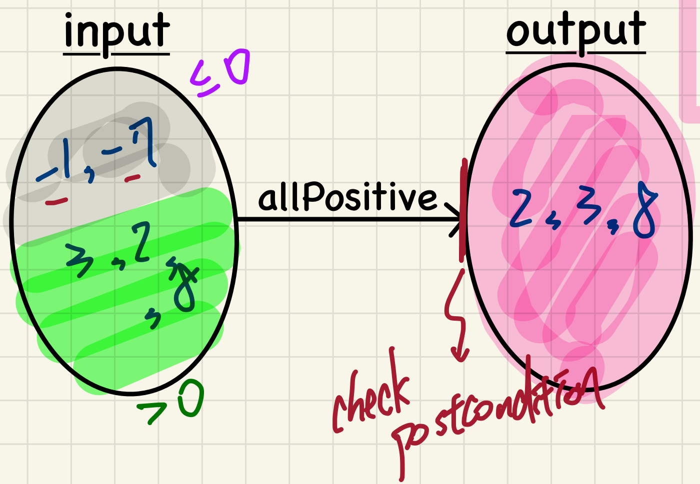
```
/* Return the set of positive elements from input. */
HashSet<Integer> allPositive(HashSet<Integer> input)
```

Say:

- S denotes the subset all positive elements from `input`.
- Set `output` denotes the return value from `allPositive`.

Relate the two sets S and **output** with set operators.

Imp. (faulty)



Postcondition checks to see if the output is **correct** w.r.t. the input.

postcond. (R1): incomplete if it evaluates to true despite the imp. being faulty.

(R1) $\{x \mid x \in input \land x > 0\} \subseteq output$

(R2) $output \subseteq \{x \mid x \in input \land x > 0\}$

entire postcond.

Q1: Why is $\boxed{\text{(R1)}}$ alone **incomplete**?

Q2: Why is (R2) alone **incomplete**?

witness:
input: {-1, -7, 3, 2, 8}
output: {3, 2, 8, 10}

$\forall \quad \exists \quad \Rightarrow$

(R1) $\{x \mid x \in \text{input} \wedge x > 0\} \subseteq \text{output}$

(R2) $\text{output} \subseteq \{x \mid x \in \text{input} \wedge x > 0\}$

faulty Imp.

input: $\{-1, -7, 3, 2, 8\}$

output: $\{2, 3, 8, 10\}$

(R1) T

(R2) F

incomplete

witness to show that R1 alone as the postcondition is incomplete.

What if (R2) alone is the postcond. ?

input: $\{-1, -7, 3, 2, 8\}$

~~output: $\{2, 1, 3\}$~~ output: $\{2, 3\}$

satisfies R2 but not R1

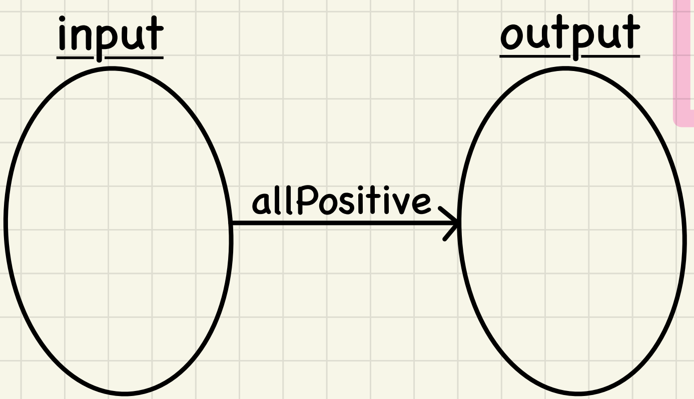
Bidirectional Subset Relations: Programming

```java
/* Return the set of positive elements from input. */
HashSet<Integer> allPositive(HashSet<Integer> input)
```

Say:

- S denotes the subset all positive elements from `input`.
- Set `output` denotes the return value from `allPositive`.

Relate the two sets S and output ~~with~~ without set operators.



Postcondition checks to see if the output is correct w.r.t. the input.

(R1) $\forall x \bullet x \in input \land x > 0 \Rightarrow x \in output$

(R2) $\forall x \bullet x \in output \Rightarrow x \in input \land x > 0$

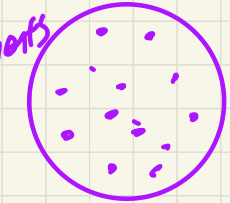
Q1: Why is (R1) alone incomplete?

Q2: Why is (R2) alone incomplete?

$\binom{n}{r}$ out of n given elements, how many ways are there to make a set of card. r

choose operator

n elements



make a seq of size r

$\underline{n}$ $\underline{(n-1)}$ $\underline{(n-2)}$ $\cdots$ $\underline{}$

1 2 3 r

eliminate duplicates

$$= \frac{n!}{(n-r)!\,r!}$$

$$= \frac{\boxed{n\cdot(n-1)\cdot(n-2)\cdot\cdots\ \_\_\_}}{r!}$$

r terms

$n-r+1$

eliminate duplicates

$\binom{4}{2}$

a b c d

$$\frac{4\times 3}{2!}$$

eliminate duplicates

ab
ba
bc
cb

$$\binom{10}{2} = \frac{10!}{(10-2)!\,2!}$$

$$= \frac{10!}{8!\,2!}$$

$$= \frac{10 * 9}{2!}$$

2 terms.

Set of Tuples

$$|S_1 \times S_2 \times \cdots \times S_n| = |S_1| \times |S_2| \times \cdots \times |S_n|$$

Given n sets $S_1, S_2, \ldots, S_n$, a ***cross/Cartesian product*** of theses sets is a set of n-tuples.

Each ***n-tuple*** $(e_1, e_2, \ldots, e_n)$ contains n elements, each of which a member of the corresponding set.

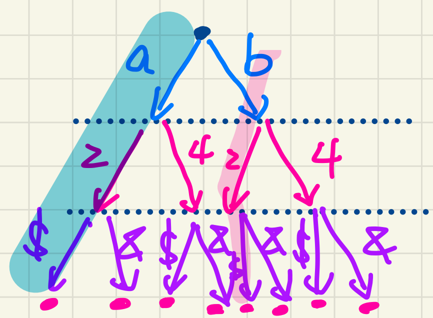
each member in the resulting set is a n-tuple.

$$\boxed{S_1 \times S_2 \cdots \times S_n} = \{\boxed{(e_1, e_2, \ldots, e_n)} \mid e_i \in S_i \land 1 \le i \le n\}$$

Example: Calculate $\underbrace{\boxed{\{a, b\}}}_{S_1} \times \underbrace{\boxed{\{2, 4\}}}_{S_2} \times \underbrace{\boxed{\{\$, \&\}}}_{S_3}$

$$= \{(e1, e2, e3) \mid e1 \in \{a, b\} \land e2 \in \{2, 4\} \land e3 \in \{\$, \&\}\}$$

$$= \{(a, 2, \$), (b, 2, \$), \ldots\}$$



Relation : set of ordered pairs

e.g. a relation on $\{1, 2, 3\}$ (S) and $\{a, b\}$ (T)...

- Is $(1, a)$ a relation on S and T?

  No! ∵ $(1, a)$ is <u>not</u> a set.

- Is $\{(\boxed{1}, \boxed{2})\}$ a relation on S and T?

  No! the order is wrong.

- Is $\{(1, a), (3, b)\}$ a relation on S and T?

$R_1 = \{(1, a), (3, b)\}$ YES!

$R_2 = \{(3, b), (1, a)\}$

$R_1 = R_2$

What's the min relation on S and T? $\emptyset$

What's the max relation on S and T? $S \times T$